# Method for reliable realization of a $\varphi$ Josephson junction

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We propose a method to realize a $\varphi$ Josephson junction by combining alternating 0 and $\pi$ parts (sub junctions) with an intrinsically nonsinusoidal current-phase relation (CPR). Conditions for the realization of the $\varphi$ ground state are analyzed. It is shown that taking into account the nonsinusoidal CPR for a "clean" junction with a ferromagnetic (F) barrier, one can significantly enlarge the domain (regime of suitable F-layer thicknesses) of the $\varphi$ ground state and make the practical realization of $\varphi$ Josephson junctions feasible. Such junctions may also have two different stable solutions, such as 0 and $\pi$, 0 and $\varphi$, or $\varphi$ and $\pi$.

## I. INTRODUCTION

The interest on Josephson junction (JJ) devices with a ferromagnetic (F) barrier has been continuously increasing during the last years.[1,2] The critical current density $j_c$ of a SFS (S, superconductor) junction exhibits damped oscillations as a function of the F-layer thickness $d_F$ so that the Josephson phase $\phi$ can be 0 or $\pi$ in the ground state.[1,2] $\pi$ junctions can be used as (nondischargeable) on-chip $\pi$-phase batteries for self-biasing various electronic circuits in the classical and quantum domains, e.g., self-biased rapid single flux quantum (RSFQ) logic[3] or flux qubits.[4,5] In addition, for quantum circuits, self-biasing also decouples the circuit from the environment and improves decoherence figures, e.g., as in the quiet qubit.[4,6,7] In classical circuits, a phase battery may also substitute the conventional inductance and substantially reduce the size of an elementary cell.[8] Some of these proposals were already realized practically.[3,9]

In this context, it is even more interesting to create a Josephson junction with an arbitrary phase difference $\varphi$ ($0 < |\varphi| < \pi$) in the ground state—a so-called $\varphi$ junction.[10–12] In addition to providing an arbitrary phase bias, such $\varphi$ junctions may have rather interesting physical properties such as two critical currents, non-Fraunhofer Josephson current dependence on an external magnetic field, half-integer Shapiro steps, and an unusual behavior when embedded in a superconducting quantum interference device (SQUID) loop. In *long* $\varphi$ junctions with degenerate ground state $\phi = \pm \varphi$, two types of mobile Josephson vortices carrying fractional magnetic flux $\Phi_1 < \Phi_0$ and $\Phi_2 = \Phi_0 - \Phi_1$ ($\Phi_0 \approx 2.07 \times 10^{-15}$ Wb is the magnetic-flux quantum) may exist, resulting in half integer zero-field steps, two critical values of magnetic field penetration, and other unusual properties.[13]

To obtain a $\varphi$ junction, the Josephson energy density

$$E_J = \frac{\Phi_0}{2\pi} \int_0^\phi j(\phi') d\phi' \qquad (1)$$

should have a local minimum at $\phi = \varphi$ ($0 < |\varphi| < \pi$). To achieve this, the current-phase relation (CPR) $j(\phi)$ should be different from the usual sinusoidal one

$$j(\phi) = j_1 \sin \phi, \qquad (2)$$

where $j$ is the Josephson (super)current density and $j_1$ is the critical current density. It has been shown that a $\varphi$ junction can be realized with a second harmonic in the CPR, i.e.,

$$j(\phi) = j_1 \sin \phi + j_2 \sin 2\phi, \qquad (3)$$

if $j_2 < 0$ and[13]

$$\left| \frac{2j_2}{j_1} \right| > 1. \qquad (4)$$

Note that in Eq. (3), $j_1$ is not anymore the critical current density, but just the amplitude of the first harmonic and $j_2$ is the amplitude of the second harmonic. The critical current density $j_c$ is then determined by local maxima of $j(\phi)$ as shown below.

A nonsinusoidal CPR is not so exotic for Josephson junctions with a ferromagnetic barrier,[2] as demonstrated recently in several experiments.[14,15] For example, in the simplest case of SFS junctions consisting of pure S and F metals, the CPR is strongly nonsinusoidal at a temperature $T \ll T_c$, where $T_c$ is the critical temperature of the S metal. Different scattering mechanisms in the F-layer also influence the CPR: usual (nonpair breaking) scattering ("dirty" limit)[16,17] spin-flip scattering,[18] and scattering of electrons from the *s* to the *d* band.[5,19] In addition, the transparency of the interfaces or the presence of an extra insulating (I) dielectric layer, such as in SIFS or SIFIS junctions, influences the CPR as well.[20–22] However, the theoretical models[2,5,17–20] that take into account different scattering mechanisms give similar results: the amplitude of the second harmonic oscillates and decreases with $d_F$ twice faster than the first one. To satisfy Eq. (4), one is tempted to choose $d_F$ in the vicinity of 0-$\pi$ transition, where $|j_1| \ll |j_2|$. Unfortunately, there $j_2 > 0$,[5,17–19] which excludes a $\varphi$ ground state.

A technique to create a negative second harmonics artificially was proposed recently.[10,23] By using one of the available 0-$\pi$ junction technologies,[24,25] one fabricates a Josephson junction with alternating short 0 ($j_1 > 0$) and $\pi$ ($j_1 < 0$) regions, which all have the simple CPR (2). Here, an *effective* second harmonic with negative amplitude is generated for the averaged phase, which is slowly varying on the scale

of the facet length.[10,23] Let the alternating 0 and $\pi$ regions have the facet lengths $a$ and $b$, respectively, so that $a \approx b$. It is assumed that $a, b < \lambda_J^a, \lambda_J^b$, where $\lambda_J^a = \lambda_J(j_1^a)$ and $\lambda_J^b = \lambda_J(j_c^b)$ are the Josephson penetration depths in the corresponding parts, with

$$\lambda_J(j_c) = \sqrt{\frac{\Phi_0}{2\mu_0 \pi d' |j_c|}}. \tag{5}$$

Here, $\mu_0 d'$ is the effective inductance per square of the junction electrodes.

The effective second harmonic has a maximum amplitude if $a = b \approx \lambda_J$ and if the values of the critical current densities in corresponding regions $j_1^a \approx -j_1^b$ (see Refs. 10 and 13). The effective second-harmonic amplitude is large enough only if $j_1^a$ and $j_1^b$ are very close by absolute values, demanding to choose thicknesses $d_{F,a}$ and $d_{F,b}$ with very high precision less than 1 Å for usual dirty SFS junctions. This precision is not achievable technologically; therefore, a controllable $\varphi$ junction is quite hard to realize in this way.

However, ferromagnetic Josephson junctions already have some intrinsic second harmonic. Thus, the question is what will be the effective CPR in multifaceted junctions if one takes into account such an intrinsic second harmonic. Usually, $j_2 < 0$ in the range of $d_F$ where $j_1$ is rather large. The idea of the method used here is the following. We use two thicknesses $d_{F,a}$ and $d_{F,b}$ where $j_2(d_{F,a}) < 0$ and $j_1(d_{F,a}) \approx -j_1(d_{F,b})$. By making a steplike F-layer changing between $d_{F,a}$ and $d_{F,b}$, we effectively cancel the large first harmonic and create an effective negative second harmonic which adds up with intrinsic second harmonic.

The paper is organized as follows. In Sec. II, we describe the model and derive the equations for the effective phase using an averaging procedure over the rapid oscillations of the phase on the length scales set by $a$ and $b$. Section III contains the discussion and results of calculations done for a "clean" SFS junction. Section IV concludes this work.

## II. MODEL

Mints and co-workers[23,26,27] considered one-dimensional junctions (along $x$ direction) with a critical current density $j_1(x)$ which is a random or periodic function changing on a length scale $a, b \ll \lambda_J^{a,b} = \lambda_J(j_{1a,b})$. Koshelev *et al.*[10,13] found an exact solution of this problem if $j_1(x)$ in Eq. (2) alternates between the constant values $j_1^a$ and $j_1^b$. Both groups assumed that $j_2 = 0$ within each region. Instead, here we assume that the CPR $j(d_F(x), \phi)$ is nonsinusoidal as a function of $\phi$ within each region and alternates between $j(d_{F,a}, \phi)$ and $j(d_{F,b}, \phi)$ as a function of $x$. Then the problem of calculating the phase distribution along such a nonuniform structure can be reduced to the well-known problem of a nonlinear oscillator behavior.[28]

The Josephson current in every region, as an odd function of the phase, can be expanded in a series of harmonics

$$j[d_F(x), \phi] = \sum_{n=1}^{\infty} j_n[d_F(x)]\sin n\phi \tag{6}$$

and the problem is solved in a general form. Here, $j_n$ denotes

the amplitude of the *intrinsic n*th harmonic of the current density.

We assume $\xi_F \ll a, b \leq \lambda_J$, where $\xi_F$ is the ferromagnetic coherence length, i.e., the characteristic length for the critical current density nonuniformity at the region of the steplike change of junction properties.[29] Let us consider one period of the structure $[-a; b)$ with F-layer thickness

$$d_F(x) = \begin{cases} d_{F,a}, & x \in [-a,0) \\ d_{F,b}, & x \in [0,b). \end{cases} \tag{7}$$

We rewrite Eq. (6) to separate the average and the oscillating parts of the Josephson current, i.e.,

$$j(x, \phi) = \sum_{n=1}^{\infty} \langle j_n \rangle [1 + g_n(x)]\sin n\phi, \tag{8}$$

where $j(x, \phi) = j[d_F(x), \phi]$. The average value of any function $f(x)$ is defined as

$$\langle f \rangle = \frac{1}{a+b} \int_{-a}^{b} f(x)dx. \tag{9}$$

Then the averaged supercurrent of the $n$th harmonic is

$$\langle j_n \rangle = \frac{aj_n(d_{F,a}) + bj_n(d_{F,b})}{a+b} \tag{10}$$

and the corresponding oscillating part is

$$g_n(x) = [j_n(x) - \langle j_n \rangle]/\langle j_n \rangle, \tag{11}$$

so that $\langle g_n \rangle = 0$ by definition. Here, $j_n(x, \phi) = j_n[d_F(x), \phi]$.

One can represent the Josephson phase $\phi(x)$ as a sum of a slow component $\psi(x)$, changing on a distance $\sim \Lambda_J = \lambda_J(\langle j_1 \rangle)$ [see Eq. (5)], and a rapid component $\zeta(x)$, changing on a distance $\sim a, b$, i.e.,

$$\phi(x) = \psi + \zeta(x). \tag{12}$$

Here we assume that the junction is short ($l \ll \Lambda_J$) and do not write the $x$ dependence for $\psi$. We also assume that the average of fast phase oscillations is vanishing, i.e.,

$$\langle \zeta \rangle = 0, \tag{13}$$

and their amplitude is small

$$\langle |\zeta| \rangle \ll 1. \tag{14}$$

The ground state of the Josephson junction is determined by the Ferrel-Prange equation

$$\Lambda_J^2 \frac{\partial^2 \phi}{\partial x^2} = \frac{j(x, \phi)}{|\langle j_1 \rangle|}. \tag{15}$$

Substituting Eqs. (8) and (12) into Eq. (15) and keeping the terms up to first order in $\zeta(x)$, we can obtain equations for the rapid phase $\zeta$,

$$\Lambda_J^2 \frac{\partial^2 \zeta}{\partial x^2} - \sum_{n=1}^{\infty} \beta_n g_n(x)\sin n\psi = 0. \tag{16}$$

To obtain the equation for the slow phase $\psi$, we average Eq. (15) over the length $(a+b) \ll \Lambda_J$ and get

$$\Lambda_J^2 \frac{\partial^2 \psi}{\partial x^2} - \sum_{n=1}^{N} [\beta_n \sin n\psi + n\beta_n \langle g_n \zeta \rangle \cos n\psi] = 0, \quad (17)$$

where $\beta_n = \langle j_n \rangle / |\langle j_1 \rangle|$. The number of harmonics $N$, which is reasonable to take into account within the given approximation, follows from the condition $N|\zeta(x)| \ll 1$. We have to find the function $\zeta(x)$ from Eq. (16), calculate average values $\langle g_n \zeta \rangle$, and substitute them into Eq. (17). Since for steplike $d_F(x)$ the Josephson current (6) is a steplike function of $x$, Eq. (16) has the form $\partial^2 \zeta / \partial x^2 = \text{const}$ on every interval $[-a; 0)$ and $[0; b)$. Its solution is a parabolic segment. The function $\zeta(x)$ must be continuous at $x=0$ and at the edges, i.e., it must satisfy the boundary condition $\zeta(-a) = \zeta(b)$. Moreover, it should satisfy Eq. (13).

It is convenient to expand the rapid phase $\zeta(x)$, as a solution of Eq. (16), into a series

$$\zeta(x) = \sum_{n=1}^{\infty} \zeta_n(x) \sin n\psi. \quad (18)$$

From this, the average values are calculated as

$$\beta_n \langle g_n \zeta_k \rangle = -\frac{2\alpha \delta j_n \delta j_k}{|\langle j_1 \rangle|}; \quad n, k = 1, 2 \dots, \quad (19)$$

where $\delta j_n \equiv j_n(d_{F,a}) - j_n(d_{F,b})$ and

$$\alpha \equiv \frac{a^2 b^2}{24 \Lambda_J^2 (a+b)^2 |\langle j_1 \rangle|}. \quad (20)$$

By definition (5), $\Lambda_J^2 |\langle j_1 \rangle| = \lambda_J^2 |j_{1a}|$, where for brevity $j_{1a} \equiv j_1(d_{F,a}), \lambda_J \equiv \lambda_J(j_{1a})$.

The dependence of the Josephson current on the slow phase, which changes on a large distance of the order of $\Lambda_J$, follows from Eq. (17) as

$$J(\psi) = |\langle j_1 \rangle| \left[ \sum_n \beta_n \sin n\psi + \sum_{n,k} n\beta_n \langle g_n \zeta_k \rangle \cos n\psi \sin k\psi \right]. \quad (21)$$

The CPR (21) contains contributions of two types: intrinsic harmonics, which are given by the first term of Eq. (21), and the effectively generated ones, which are given by the second term. Intrinsic contributions are defined by the CPR (6) of the junction regions, while generated contributions effectively appear as a result of averaging over fast oscillations.

The amplitudes of the effectively generated harmonics, which are proportional to average values (19), are largest by absolute value if $\alpha$ and $|\delta j_n|$ reach their maximum. This happens if the lengths of $a$ and $b$ facets have the largest possible size which still allows averaging, i.e.,

$$a = b \approx \lambda_J \quad (22)$$

and

$$j_n(d_{F,a}) \approx -j_n(d_{F,b}). \quad (23)$$

Condition (23) ensures $|\langle j_1 \rangle| \ll |j_{1a}|$ and consequently $\Lambda_J \gg \lambda_J$. It was shown[10] that even if the equality (22) holds exactly, condition (14) is satisfied. In this case, $\alpha = 1/96|j_{1a}|$ [cf. Eq. (20)].

In all theoretical models developed up to now,[2,5,17–20] the second harmonic was usually considered to be much smaller than the first one (except for the points of the $0$-$\pi$ transition, where $j_1 \to 0$), with an even smaller third harmonic. So, the expression (6) can be considered as a Taylor expansion in some small parameter. Then, keeping terms of the same order of this small parameter, we obtain $n+k=N+1$ effective harmonics in the CPR of the multifaceted $0$-$\pi$ junction.

The number $N$ of harmonics, which is reasonable to take into account, follows from the estimate of the short-range phase $N \max|\zeta(x)| \ll 1$. As $\zeta(x)$ has a parabolic form, it takes its maximum absolute value at the center of every interval $-a/2$ and $b/2$. With the estimate

$$\sum_{n=1}^{\infty} \beta_n g_n(x) \sin n\psi \sim g_1, \quad (24)$$

one obtains

$$|\zeta(-a/2)| \sim \frac{ab(a+2b)|\delta j_1|}{24 \Lambda_J^2 (a+b)|\langle j_1 \rangle|}. \quad (25)$$

This is maximized if $a=b=\lambda_J$. Since $\max|\delta j_1| \approx 2|j_{1a}|$, we estimate $\max|\zeta(x)| \approx 1/8$. Thus, as a reasonable choice one can take $N=3$. So, within the above approximations, it is reasonable to consider three intrinsic harmonics with amplitudes $j_{1\dots3}$ and four generated harmonics, i.e.,

$$J(\psi) = \sum_{n=1}^{4} J_n \sin n\psi, \quad (26)$$

with

$$J_1 = \langle j_1 \rangle + \alpha \delta j_1 \delta j_2,$$

$$J_2 = \langle j_2 \rangle - \alpha \delta j_1^2 + 2\alpha \delta j_1 \delta j_3,$$

$$J_3 = \langle j_3 \rangle - 3\alpha \delta j_1 \delta j_2,$$

$$J_4 = -2\alpha \delta j_2^2 - 4\alpha \delta j_1 \delta j_3. \quad (27)$$

Here, $J_n$ denotes the total amplitude of the $n$th harmonic in the CPR for the averaged phase. These expressions reduce to the earlier results[10,13] if one takes into account only the intrinsic first harmonic. The expression for every harmonic is a simple sum of the corresponding average intrinsic harmonic and the generated part.

## III. RESULTS AND DISCUSSION

We first address the question which type of ferromagnetic junction with $0$-$\pi$ facets can satisfy the conditions for a $\varphi$ junction in the best way. The critical current density $j_1$ of a Josephson junction with a ferromagnetic barrier having a complex coherence length $\xi_F (\xi_F^{-1} = \xi_1^{-1} + i\xi_2^{-1})$ decays $\propto \exp(-d_F/\xi_1)$ and oscillates with a period[2] $2\pi\xi_2$ in the dirty limit (electron mean-free path $l \ll \xi_{1,2}$). The decay length $\xi_1$ depends on $l$ as well as on a pair-breaking scattering length in the ferromagnet (spin-flip scattering[18] or scattering into the $d$ band;[30] both influence the CPR). Pair-breaking scattering

leads to[19,31] $\xi_1 < \xi_2$ and the Josephson current decays so rapidly in the dirty limit that there are only very tiny regions of $d_{F,a}$ and $d_{F,b}$ on the dependence $j_1(d_F)$, where $j_1(d_{F,a}) \approx -j_1(d_{F,b})$. In the clean limit, the Josephson current decays much more slowly with increasing $d_F$. The cleaner is the ferromagnet, the larger is $l$, and the slower is the decay [cf. Eqs. (23) and (24) with Eq. (7) from Ref. 17 or see the discussion in Ref. 30]. In the limit $l \gg (d_F, \xi_{1,2})$, the critical current density decreases as[16] $1/d_F$. Moreover, a clean SFS junction has a nonsinusoidal CPR and its second harmonics $j_2 < 0$ in some regions of $d_F$ far from 0-$\pi$ transitions, which can effectively help to satisfy the conditions for the realization of a $\varphi$ junction. It was shown in different models that the second harmonic $j_2(d_F)$ decays and oscillates with $d_F$ twice faster than $j_1(d_F)$. Therefore, it is reasonable to take an F-layer with $d_{F,a}$ and $d_{F,b}$ of the order of a few $\xi_2$. Usually, $\xi_2 = v_F/2E_{ex}$, where $v_F$ is the Fermi velocity and $E_{ex}$ is the exchange magnetic energy in the ferromagnet.[17,30,32] It was established experimentally that for pure ferromagnetic metals, $\xi_2$ is largest for Ni.[33–36] Higher harmonics also decay rapidly with increasing temperature.[17,37] We have also checked this statement for models described in Refs. 16 and 19. It is clear that by approaching $T_c$ from below the superconducting gap $\Delta \to 0$, equations become linear and their simple exponential solutions yield only a sinusoidal CPR. Therefore, the most promising strategy for realization of a $\varphi$ junction is to use a pure SFS junction with a thin Ni layer at low temperature ($\lesssim 0.1 T_c$). The model describing the Josephson effect in such junctions was established long ago.[16] It is based on the solution of the Eilenberger equations.

If the ferromagnetic film has magnetization component perpendicular to the plane of the junction barrier, we can ignore its influence on the phase difference because the phase gradient is a consequence of the in-plane component of the magnetic field. We can neglect this effect also for in-plane magnetization components of the ferromagnetic film in the case, when the size $L_d$ of magnetic domains $\xi_{1,2} \ll L_d \ll \lambda_J$. The corresponding phase variations would be much shorter in space than for the considered short-range phase and their contribution would be of the order of $L_d^2/\lambda_J^2 \ll ab/\lambda_J^2$ by our assumption $a \sim b \sim \lambda_J$. Crossed Andreev reflection (in the clean limit) or a triplet pairing may arise in the area of domain walls and produce a long-range triplet component of the Josephson current.[38,39] This triplet component is not significant for small values of $d_F$ where the usual singlet current dominates. An experiment[31] with Nb|CuNi|Nb junctions demonstrated the absence of a significant influence of a domain structure on CPR for multidomain samples[40] (see classical Fraunhofer dependence in Fig. 3 of Ref. 31).

What is the measurable critical current density $J_C$ (maximum supercurrent) of a Josephson junction with a nonsinusoidal CPR? It is not anymore $|j_1| = |j(\pi/2)|$ as follows from Eq. (2), but the local maximum of the expression (6) with respect to $\phi$. Note that Eq. (6) may allow several local extrema in the interval $[0, 2\pi]$.

Figure 1(a) shows the three first harmonics vs F-layer thickness and Fig. 1(b) shows the corresponding $J_C(d_F)$ for the model of an SFS junction as described in Ref. 16 (see also Ref. 17). It is interesting to note that near a 0-$\pi$ transition, when the first harmonic is small, $j(\phi)$ may have two
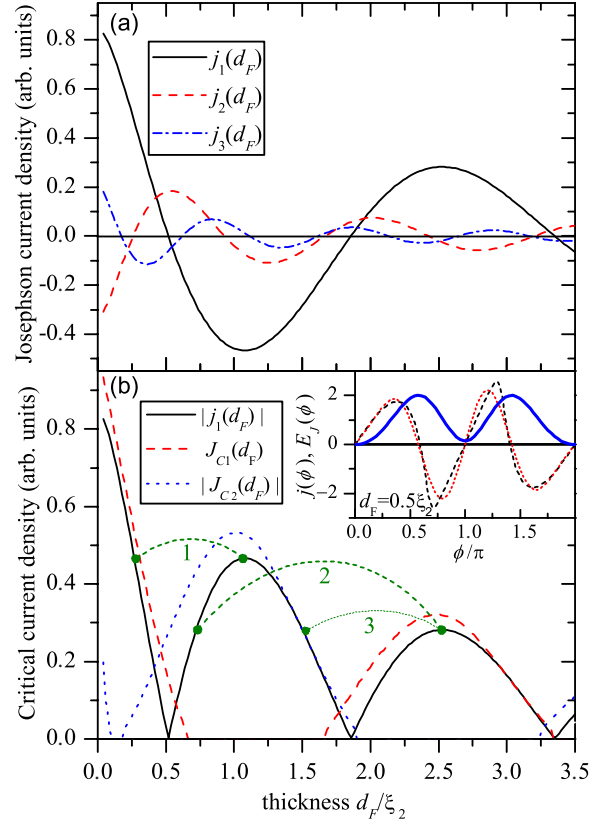


FIG. 1. (Color online) Properties of a uniform SFS Josephson junction in the clean limit at $T = 0.1 T_c$. (a) Amplitudes of the first three harmonics $j_1, j_2, j_3$ of Eq. (6) as a function of the ferromagnet thickness $d_F$. (b) Measurable critical current densities $J_{C1}$ and $|J_{C2}|$ vs $d_F$, which are realized at the phase $0 < \phi < \pi$ and $\pi < \phi < 2\pi$, respectively. Inset shows for $d_F = 0.5\xi_2$ the CPR $j(\phi)$ from Eq. (3) in Ref. 16 (dashed line) and from the approximation by the first three harmonics (dotted line), with the corresponding $E_J(\phi)$ dependence (solid line). For comparison, in (b), the critical current $|j_1(d_F)|$ of a junction with only the first harmonic in the CPR is also shown. Here, pairs of points (connected by arcs) where $j_1(d_{F,a}) = -j_1(d_{F,b})$ correspond to a $\varphi$ ground state of a multifaceted SFS junction with alternating thicknesses $d_{F,a}, d_{F,b}$ [see areas 1–3 in Fig. 2(a)].

different local maxima and the measurable $J_C(d_F)$ has two different values $J_{C1}, |J_{C2}|$ depending on the initial state of the junction[13] [see Fig. 1(b)]. An example of a CPR $j(\phi)$ with two different maxima and the corresponding Josephson energy $E_J(\phi)$ (1) near a 0-$\pi$ transition are presented in Fig. 1(b) (inset). Thus, even for a uniform Josephson junction (where the $\varphi$-ground state is impossible) close to a 0-$\pi$ transition, two different values of critical current density could be realized if its CPR differs enough from the sinusoidal one.

In the framework of the clean SFS junction model,[16] we investigate a multifaceted junction with $a = b = \lambda_J \ll \Lambda_J$. For $\lambda_J$, we take the value corresponding to the first minimum of $j_1$ at $d_F \approx \xi_2$ [see Fig. 1(a)]. For clean SFS junctions, this value is[35] $\approx 10$ kA/cm$^2$, which corresponds to $\lambda_J \approx 3$ $\mu$m and which allows to realize $a \approx b$ with reasonable precision. Below we investigate the ground states in a Josephson junction with alternating regions of lengths $a$ and $b$ and F-layer thicknesses $d_{F,a}$ and $d_{F,b}$ varying from 0 to a few $\xi_2$.
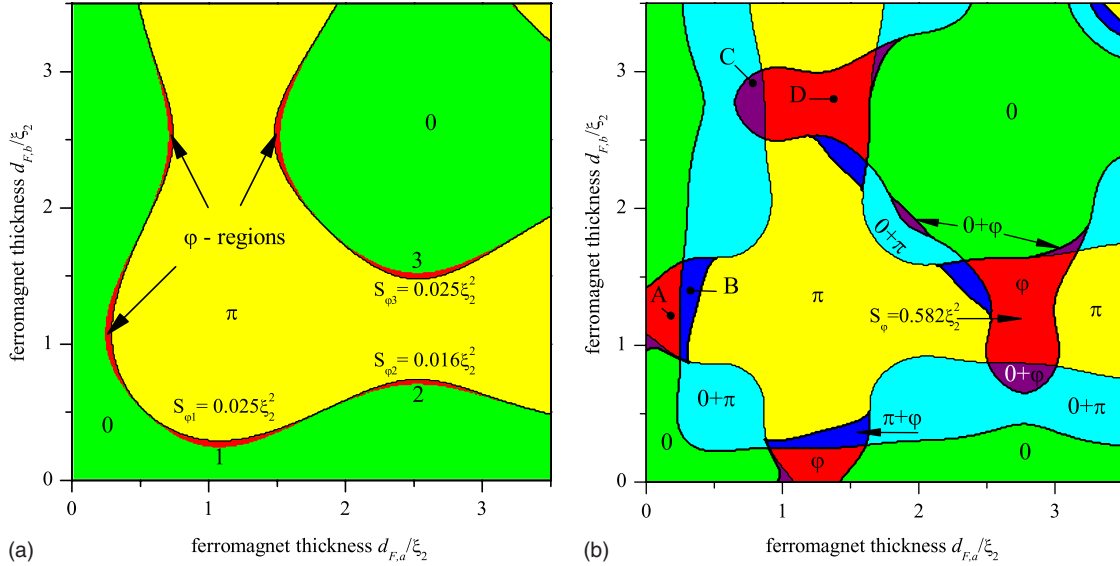
FIG. 2. (Color online) Ground states scheme for the multifaceted SFS junction with F-layer thickness periodically changing between $d_{F,a}$ and $d_{F,b}$ plotted on the $(d_{F,a}, d_{F,b})$ plane. The values of the ground state phase corresponding to each region are shown and the areas $S_\varphi$ for some of the different $\varphi$ ground state regions are also indicated. In (a), the calculation includes only the first harmonics in every uniform part, i.e., two generated harmonics. The $\varphi$ regions 1, 2, and 3 in (a) correspond to the pairs of $d_{F,a}$ and $d_{F,b}$ that are shown by arcs in Fig. 1(b). In (b), the first three harmonics and the corresponding four generated harmonics are taken into account [see Eqs. (26) and (27)]. The dependence $E_J(\psi)$ for points A–D is shown in Fig. 3.

The ground state corresponds to a local minimum of the energy (1) with the CPR (21). The junction has a stable static solution $\psi = 0$ (0 phase) if

$$\sum_{n=1}^{N} \left[ n\langle j_n \rangle - \alpha \sum_{k=1}^{N+1-n} nk \delta j_n \delta j_k \right] > 0 \tag{28}$$

and the solution $\psi = \pi$ ($\pi$ phase) if

$$\sum_{n=1}^{N} \left[ n\langle j_n \rangle - \alpha \sum_{k=1}^{N+1-n} (-1)^{n+k} nk \delta j_n \delta j_k \right] > 0. \tag{29}$$

For the CPR (26), these conditions have the following form: for the 0 phase

$$J_1 + 2J_2 + 3J_3 + 4J_4 > 0 \tag{30}$$

and for the $\pi$ phase

$$J_1 - 2J_2 + 3J_3 - 4J_4 < 0. \tag{31}$$

Both solutions $\psi = 0$ and $\psi = \pi$ coexist when these conditions are satisfied simultaneously, i.e.,

$$2J_2 + 4J_4 > |J_1 + 3J_3|. \tag{32}$$

If both conditions (30) and (31) are not satisfied, i.e.,

$$2J_2 + 4J_4 < -|J_1 + 3J_3|, \tag{33}$$

only the $\varphi$ ground state is possible. The conditions (30)–(33) coincide with the conditions (4) if $J_3 = J_4 = 0$. Generally, the $\varphi$ junction is realized if the Josephson energy $E_J(\psi)$ (1) has a local minimum, i.e., if the CPR $J(\psi)$ (26) crosses $J = 0$ from a negative to a positive value at some point $\psi = \varphi \neq 0, \pi$. This is the case if the equation

$$J(\psi) \sim 8J_4 z^3 + 4J_3 z^2 + (2J_2 - 4J_4)z + J_1 - J_3 = 0 \tag{34}$$

has at least one real solution $z = \cos \psi$, satisfying the conditions $|z| < 1$ (which gives $0 < |\psi| < \pi$) and

$$\frac{\partial J}{\partial \psi} \sim 32J_4 z^4 + 12J_3 z^3 + (4J_2 - 8J_4)z^2 + (J_1 - 9J_3)z - 2J_2$$
$$+ 4J_4 > 0, \tag{35}$$

which ensures the local minimum of $E_J(\psi)$. Starting from a pair of $d_{F,a}, d_{F,b}$, we calculate $J_{1\ldots4}$. Then from Eqs. (30)–(35), the possible ground states are identified. The resulting phase diagram is shown in Fig. 2, where different ground states for each pair of $d_{F,a}, d_{F,b}$ are shown by different colors.

Figure 2(a) shows the results obtained if only the first intrinsic harmonic is taken into account. Here, the areas of 0 and $\pi$ ground state phase are separated by slim regions of $\varphi$ phase. It is clear that the phase diagram is symmetric with respect to the line $d_{F,a} = d_{F,b}$. Therefore, below, without loosing generality, we focus on the case $d_{F,a} > d_{F,b}$. In the chosen interval of thicknesses, there are three areas of $\varphi$ phase, marked as 1, 2, and 3. In these areas $j_1(d_{F,a}) \approx -j_1(d_{F,b})$, which corresponds to the pairs of $d_{F,a}$ and $d_{F,b}$ shown in Fig. 1(b) as 1, 2, and 3. If $d_{F,a}$ and $d_{F,b}$ are not well controllable, the area $S_\varphi$ of $\varphi$ regions is proportional to the probability of the $\varphi$ junction realization. For all three different cases shown in Fig. 2(a), $S_\varphi$ is rather small ($\leq 0.025\xi_2^2$). Hence, one has to control $d_{F,a}$ and $d_{F,b}$ extremely precisely in order to realize a $\varphi$ junction. In the case of a dirty SFS junction, when $j_1(d_F)$ decays exponentially, the area $S_\varphi$ is even smaller and the probability to fabricate a $\varphi$ junction is vanishing.
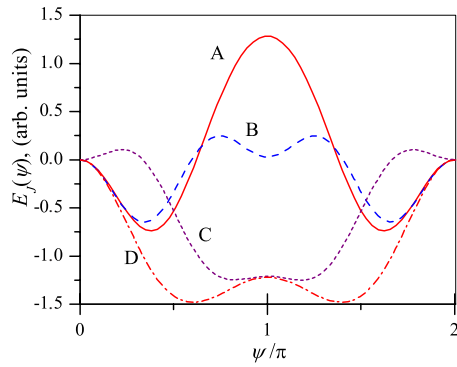
FIG. 3. (Color online) Josephson energy $E_J(\psi)$ of multifaceted SFS junctions for different combinations of $F$-layer thicknesses $d_{F,a}$ and $d_{F,b}$ as indicated by points A–D in Fig. 2(b). A and D correspond to the $\varphi$ ground state, B corresponds to two stable states $\pi$ and $\varphi$, and C corresponds to two stable states 0 and $\varphi$.

However, if we take into account a nonsinusoidal CPR for the clean SFS junction, the areas of the $\varphi$ ground state become much larger as can be seen in Fig. 2(b). This is a consequence of the fact that the intrinsic second harmonic $j_2(d_F) < 0$ in the corresponding regions, which efficiently helps to make the absolute value of the generated second harmonic large enough.

In particular, the areas 2 and 3 from Fig. 2(a) merge and form a compact $\varphi$ ground state region with area $S_\varphi \approx 0.6\xi_2^2$ around $d_{F,a} \sim 2.8\xi_2$ and $d_{F,b} \sim 1.4\xi_2$. This region seems to be very well suited for the experimental realization of a $\varphi$ junction, as it does not demand to produce an extremely thin F-layer and at the same time allows for reasonably large tolerances in sample fabrication. Taking the ferromagnetic coherence length $\xi_2 \sim 1.2$ nm from experiments,[34,35] the linear size of the $\varphi$ region in Fig. 2(b) is $\Delta d_{F,a} \sim \Delta d_{F,b} \sim 0.5\cdots1$ nm. Modern technology allows the control of $d_F$ with such precision.[41–43]

Finally, we note that the structure considered here may have simultaneously two different stable static solutions, 0 and $\pi$, 0 and $\varphi$, or $\pi$ and $\varphi$, as can be seen in Fig. 2(b). The corresponding $E_J(\psi)$ curves are presented in Fig. 3, where one can see several local minima of the Josephson energy as a function of the phase. Probably, taking into account the next harmonics, it is also possible to find two different $\varphi$ ground states. However, we expect the corresponding regions within the $d_{F,a}, d_{F,b}$ plane to be so small that it will be quite hard to fabricate such a structure.

## IV. CONCLUSION

We propose a method to realize a $\varphi$ junction, i.e., a junction with the Josephson phase $\varphi \neq 0$ or $\pi$ in the ground state, based on SFS junctions. In a uniform SFS Josephson junction near a 0-$\pi$ transition (with appropriate F-layer thickness $d_F$) the first harmonic $j_1 \rightarrow 0$, while the second harmonic $j_2$ dominates, however usually with positive sign ($j_2 > 0$), which excludes the formation of a $\varphi$ ground state. Instead, the main idea of the method used here to make a $\varphi$ junction is the following. We choose a thickness $d_F$ where the second harmonic is large and negative. To cancel the first harmonic, we use a periodic steplike modulation of the F-layer thickness between $d_{F,a}$ and $d_{F,b}$. Here, $d_{F,a}$ and $d_{F,b}$ are chosen such that for both of them, the second harmonic is negative, while the first harmonic is $j_1(d_{F,a}) \approx -j_1(d_{F,b})$. Periodic modulation not only cancels the first harmonic but also generates an additional negative second harmonic for the average phase. This effect can be made stronger if one works with a clean ferromagnetic barrier and uses alternating regions of equal length $a \approx b \sim \lambda_J$.

Different mechanisms, leading to a significantly nonsinusoidal current-phase relation of SFS junctions, are analyzed. The CPR mostly different from the sinusoidal one is obtained for a clean SFS junction at low temperature. In this case, there are reasonably large regions of thicknesses $d_{F,a}$ and $d_{F,b}$ (in comparison to the description taking into account only the first harmonic) where the multifaceted Josephson junction has a $\varphi$ ground state. Moreover, for some values of $d_{F,a}$ and $d_{F,b}$, such structures may have two different ground states (two local minima of the Josephson energy as a function of the phase): 0 and $\pi$, 0 and $\varphi$, or $\pi$ and $\varphi$. Our analysis gives some practical recommendations for the fabrication of SFS junctions with arbitrary phase shifts $\varphi$ in the ground state.

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